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ENERGY CONDITIONS CONSTRAINTS ON A CLASS OF $f(R)$ GRAVITY

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We present and discuss the bounds from the energy conditions on a general $f(R)$ functional form in the framework of metric variational approach. As a concrete application of the energy conditions to locally homogeneous and isotropic $f(R)$ –cosmology, the recent estimated values of the deceleration and jerk parameters are used to examine the bounds from the weak energy condition on the free parameter of the family of $f(R) = \sqrt{R^2 - R_0^2}$ gravity theory.

Keywords: Energy conditions constraints; $f(R)$ gravity theories.

1. Introduction

The observed late-time acceleration of the Universe poses a great challenge to modern cosmology, which may be the result of unknown physical processes involving either modifications of gravitation theory or the existence of new fields in high energy physics. This latter route is most commonly used, however, following the former, an attractive approach to this problem, known as $f(R)$ –gravity,¹ examines the possibility of modifying Einstein’s general relativity (GR) by adding terms proportional to powers of the Ricci scalar R to the Einstein-Hilbert Lagrangian (see

also Refs. 2 for recent reviews). Although these theories provide an alternative way to explain the observed cosmic acceleration without dark energy, the freedom in the choice of different functional forms of $f(R)$ gives rise to the problem of how to constrain on theoretical and/or observational grounds, the many possible $f(R)$ –gravity theories. Theoretical limits have long been discussed in Refs. 3, while only recently observational constraints from several cosmological datasets have been explored for testing the viability of these theories.⁴

Additional constraints to $f(R)$ theories may also arise by imposing the so-called energy conditions.⁵ It is well known that these conditions, initially formulated in GR context,⁶ have been used in different contexts to derive general results that hold for a variety of physical situations.⁷ More recently, several authors have employed the GR classical energy conditions to investigate cosmological issues such as the phantom fields⁸ and the expansion history of the universe.⁹ While they are well founded in the context of GR, one has to be cautious when using the energy conditions in a more general framework, such as the $f(R)$ –gravity. In this regard, in a recent work Santos *et al.*¹⁰ have used Raychaudhuri's equation along with the requirement that gravity is attractive, to derive the energy conditions for a general $f(R)$ –gravity in the metric formulation. They have shown that, although similar, the energy conditions differ from their formulation in GR context. In this work, we use estimated values of the deceleration and jerk parameters to examine the bounds from these newly derived $f(R)$ –energy-conditions on the one-parameter family of a recently proposed $f(R)$ –gravity.^{11,12}

2. Energy conditions in $f(R)$ gravity

The generalized action that defines an $f(R)$ –gravity is given by

$$S = \int d^4x \sqrt{-g} f(R) + S_m \quad (1)$$

where $g = \det(g_{\mu\nu})$, R is the Ricci scalar, S_m is the standard action for the matter fields, and hereafter we use units such that $8\pi G = c = 1$. Varying this action with respect to the metric $g_{\mu\nu}$ we obtain the field equations

$$f' R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) f' = T_{\mu\nu}, \quad (2)$$

where a prime denotes differentiation with respect to R and $\square \equiv g^{\alpha\beta} \nabla_\alpha \nabla_\beta$. Following and extending the GR approach, Santos *et al.*¹⁰ used the Eq. (2) together with the Raychaudhuri equation and attractiveness of gravity to show that for the Friedmann-Lemaître-Robertson-Walker (FLRW) geometry with scale factor $a(t)$, the null energy condition (NEC) and the strong energy condition (SEC) in $f(R)$ –gravity theories can be written in the form

$$\text{NEC} \implies \rho + p \geq 0, \quad \text{and} \quad (\ddot{R} - \dot{R}H) f'' + \dot{R}^2 f''' \geq 0, \quad (3)$$

$$\text{SEC} \implies \rho + 3p - f + Rf' + 3(\ddot{R} + \dot{R}H) f'' + 3\dot{R}^2 f''' \geq 0, \quad (4)$$

where a dot denotes derivative with respect to time and $H = \dot{a}/a$ is the Hubble parameter. They also have shown that, in addition to Eq. (3), the weak energy condition (WEC) requires

$$\mathbf{WEC} \implies \rho + \frac{1}{2}(f - Rf') - 3\dot{R}Hf'' \geq 0, \quad (5)$$

whereas the dominant energy conditions (DEC) fulfillment, besides the inequalities (3) and (5), demands

$$\mathbf{DEC} \implies \rho - p + f - Rf' - (\ddot{R} + 5\dot{R}H)f'' - \dot{R}^2 f''' \geq 0. \quad (6)$$

As one may easily check, for $f = R$, the well-known forms for the NEC ($\rho + p \geq 0$) and SEC ($\rho + 3p \geq 0$) in the context of GR can be recovered from Eqs. (3) and (4), while Eqs. (5) and (6) give $\rho \geq 0$ and $\rho - p \geq 0$, whose combination with Eqs (3) give, respectively, the well-known forms of the WEC and DEC in GR (see, e.g., Refs. 6 and 9).

3. Constraining $f(R) = \sqrt{R^2 - R_0^2}$ theory

As shown by Santos *et al.*¹⁰, the energy-conditions inequalities (3), (4), (5) and (6) can be used to place bounds on a given $f(R)$. In the context of FLRW models the bounds can also be stated in terms of the deceleration (q), jerk (j) and snap (s) parameters. To this end, we note that the Ricci scalar and its derivatives can be expressed as

$$R = -6H^2(1 - q), \quad (7)$$

$$\dot{R} = -6H^3(j - q - 2), \quad (8)$$

$$\ddot{R} = -6H^4(s + q^2 + 8q + 6), \quad (9)$$

where

$$q = -\frac{1}{H^2} \frac{\ddot{a}}{a}, \quad j = \frac{1}{H^3} \frac{\ddot{\ddot{a}}}{a}, \quad s = \frac{1}{H^4} \frac{\ddot{\ddot{\ddot{a}}}}{a}, \quad (10)$$

and $H = \dot{a}/a$ is the Hubble parameter. Thus, in terms of the present-day values (denoted by the subscript 0) of the above parameters the **NEC** [Eq. (3)], **SEC** [Eq. (4)], **DEC** [Eq. (6)] and **WEC** [Eq. (5)] can be, respectively, rewritten as

$$\begin{aligned} \rho_0 + p_0 \geq 0 \quad \text{and} \quad -[s_0 - j_0 + (q_0 + 1)(q_0 + 8)]f_0'' + 6[H_0(j_0 - q_0 - 2)]^2 f_0''' \geq 0, \\ \rho_0 + 3p_0 + f_0 - 6H_0^2(1 - q_0)f_0' - 6H_0^4(s_0 + j_0 + q_0^2 + 7q_0 + 4)f_0'' + 3[6H_0^3(j_0 - q_0 - 2)]^2 f_0''' \geq 0, \\ \rho_0 - p_0 + 6H_0^2(1 - q_0)f_0' - 6H_0^4[s_0 + (q_0 - 1)(q_0 + 4) + 5j_0]f_0'' - [6H_0^3(j_0 - q_0 - 2)]^2 f_0''' \geq 0, \\ 2\rho_0 + f_0 + 6H_0^2(1 - q_0)f_0' + 36H_0^4(j_0 - q_0 - 2)f_0'' \geq 0, \end{aligned} \quad (11)$$

which is in the appropriate form to confront with observations by using the estimate values of the deceleration, jerk and snap parameters.

4 Janilo Santos, Marcelo J. Rebouças, Jailson S. Alcaniz

Recently Baghran *et al.*¹¹ and Movahed *et al.*¹² have proposed a modified $f(R)$ –gravity by choosing the geometric part of the Lagrangian (1) as

$$f(R) = \sqrt{R^2 - R_0^2}, \quad (12)$$

where R_0 is a parameter to be adjusted by observations.^a One features of this theory, is that it has an intrinsic minimum spatial curvature, which provides a late time accelerating cosmic expansion. Besides, its appealing mono-parameter form makes it more tractable when testing for cosmological and solar system observations. Comparing the observed perihelion precession of Mercury with the predictions for this theory by solving the equations (2) for a spherically symmetric Schwarzschild-type metric, Baghran *et al.*¹¹ have put an upper limit $R_0 < H_0^2$.

In order to examine how the energy conditions can be used to place bounds on the free parameter R_0 of Eq. (12), we first note that, apart from the WEC [Eq. (11)], all above inequalities depend on the current value of the snap parameter s_0 . Therefore, since no reliable measurement of this parameter has been reported hitherto, we shall focus on the **WEC** requirement [given by Eq. (11)] in the confrontation of the energy condition bounds on the $f(R)$ –theory (12) with observational data. For a negligible value of the present-day density ρ_0 , a straightforward calculation shows that the **WEC**-fulfillment inequality leads to the following constraint on the free parameter of the $f(R)$ –theory (12):

$$R_0 \geq 6H_0^2 (q_0^2 - 3q_0 - 1 + j_0)^{1/2}. \quad (13)$$

Now, taking $q_0 = -0.81 \pm 0.14$ and $j_0 = 2.16_{-0.75}^{+0.81}$, as given in Ref. 13, we find that $R_0 \geq 12.36H_0^2$, which is not consistent with the above-mentioned bound $R_0 < H_0^2$. In this way the bound from the perihelion precession of Mercury leads to a violation of the **WEC**.^b

4. Final Remarks

$f(R)$ –gravity provides an alternative way to explain the current cosmic acceleration with no need of invoking either the existence of an extra spatial dimension or an exotic component of dark energy. However, the arbitrariness in the choice of different functional forms of $f(R)$ gives rise to the problem of how to constrain the many possible $f(R)$ –gravity theories on physical grounds. In this work, following the Ref.10, we have showed how to put constraints on general $f(R)$ –gravity from the energy conditions in the context of locally homogeneous and isotropic $f(R)$ –cosmology. In particular, for a negligible present-day density ρ_0 , the lower bound $R_0 \geq 12.36H_0^2$ arise from the fulfillment of **WEC** for the $f(R)$ –theory given by Eq. (12). This

^aNote that R_0 is not the present-day value of the curvature scalar R .

^bWe note that the **WEC**-fulfillment bound on R_0 is not very sensitive precision of the parameters q_0 and j_0 , so future more precise estimates of these parameters will not change considerably this bound.

WEC-fulfillment bound, however, is not consistent with the bound on R_0 from the perihelion precession of Mercury, making clear the violation of the **WEC** in this case.

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6 Janilo Santos, Marcelo J. Rebouças, Jailson S. Alcaniz

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